$$\sin(-\theta) = \boxed{\qquad\qquad} \qquad \cos(-\theta) = \boxed{\qquad\qquad}$$

$$\int \frac{1}{x^2+1}\,dx = \boxed{\qquad\qquad} \qquad \int \frac{x}{x^2+1}\,dx = \boxed{\qquad\qquad}$$

【 練習問題 53 】

(1)[*1] $\displaystyle\int_0^2 (4x^3 - 3x^2 + 5x + 3)\,dx$

Step 1. $\displaystyle\int (4x^3 - 3x^2 + 5x + 3)\,dx \quad = \quad \boxed{\qquad\qquad}$

Step 2.

$$\int_0^2 (4x^3 - 3x^2 + 5x + 3)\,dx = \Big[\ \boxed{\qquad\qquad}\ \Big]_0^2$$

$$= \boxed{\qquad\qquad} - \boxed{\qquad\qquad}$$

$$= \boxed{\qquad\qquad}$$

---

(2) $\displaystyle\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \sin x\,dx$

Step 1. $\displaystyle\int \sin x\,dx \quad = \quad \boxed{\qquad\qquad}$

Step 2.

$$\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \sin x\,dx = \Big[\ \boxed{\qquad\qquad}\ \Big]_{\frac{\pi}{3}}^{\frac{\pi}{2}}$$

$$= \boxed{\qquad\qquad} - \boxed{\qquad\qquad}$$

$$= \boxed{\qquad\qquad}$$

[*1] 改題

(3) $\displaystyle\int_0^{\frac{1}{2}} \frac{1}{\sqrt{1-x^2}}\,dx$

Step 1. $\displaystyle\int \frac{1}{\sqrt{1-x^2}}\,dx \quad = \quad \boxed{}$

Step 2.

$$\int_0^{\frac{1}{2}} \frac{1}{\sqrt{1-x^2}}\,dx = \Big[\ \boxed{}\ \Big]_0^{\frac{1}{2}}$$

$$= \boxed{} - \boxed{}$$

$$= \boxed{}$$

---

(4) $\displaystyle\int_{-1}^{\sqrt{3}} \frac{1}{1+x^2}\,dx$

Step 1. $\displaystyle\int \frac{1}{1+x^2}\,dx \quad = \quad \boxed{}$

Step 2.

$$\int_{-1}^{\sqrt{3}} \frac{1}{1+x^2}\,dx = \Big[\ \boxed{}\ \Big]_{-1}^{\sqrt{3}}$$

$$= \boxed{} - \boxed{}$$

$$= \boxed{}$$

定積分の置換積分法

【 例題 66 】

(1) $\displaystyle\int_0^1 (2x+1)^3\,dx$

$t = \boxed{}$ とおくと $dx = \boxed{}\,dt$

x	$\to$
t	$\to$

$$\int_0^1 (2x+1)^3\,dx = \int_{\text{—}}^{\text{—}} \boxed{}\,dt$$

$$\left(\int \boxed{}\,dt = \boxed{} \text{ より}\right)$$

$$= \Big[\boxed{}\Big]_{\text{—}}^{\text{—}}$$

$$= \boxed{}$$

(2) $\displaystyle\int_0^2 \sqrt{x+1}\,dx$

$t = \boxed{}$ とおくと $dx = \boxed{}\,dt$

x	$\rightarrow$
t	$\rightarrow$

$$\int_0^2 \sqrt{x+1}\,dx = \int_{\_\_}^{\_\_} \boxed{}\,dt$$

$$\left(\int \boxed{}\,dt = \boxed{} \text{ より}\right)$$

$$= \Big[\boxed{} \Big]_{\_\_}^{\_\_}$$

$$= \boxed{}$$

■ $x = a\sin\theta$ で置換積分

(3) $\displaystyle\int_0^3 \sqrt{9-x^2}\,dx$

$x = \boxed{\qquad\qquad}$ とおくと $dx = \boxed{\qquad\qquad}\,d\theta$

☞ $\sqrt{a^2-x^2}$ の時は $x = a\sin\theta$ と置くとうまくいく場合が多い.

練習問題 53(3) で確かめてみよ．(この置換積分は高校の数学 III の内容である)

x	$\rightarrow$
θ	$\rightarrow$

$$\sqrt{9-x^2} = \boxed{\qquad\qquad\qquad\qquad}$$

$$\int_0^3 \sqrt{9-x^2}\,dx = \int_{\_}^{\_} \boxed{\qquad\qquad}\,d\theta$$

$$\left(\int \boxed{\qquad\qquad}\,d\theta = \boxed{\qquad\qquad} \text{ より}\right)$$

$$= \Big[\boxed{\qquad\qquad}\Big]_{\_}^{\_}$$

$$= \boxed{\qquad\qquad}$$

■ 番外編 $x = \tan\theta$ で置換積分

$$\int_{-1}^{\sqrt{3}} \frac{1}{1+x^2}\,dx$$

$x = \boxed{\qquad\qquad}$ とおくと $dx = \boxed{\qquad\qquad}\,d\theta$

x	$\to$
θ	$\to$

$$\int_{-1}^{\sqrt{3}} \frac{1}{1+x^2}\,dx = \int_{\_\_}^{\_\_} \boxed{\qquad\qquad}\,d\theta$$

$$\left(\int \boxed{\qquad\qquad}\,d\theta = \boxed{\qquad\qquad} \text{ より}\right)$$

$$= \Big[\boxed{\qquad\qquad}\Big]_{\_\_}^{\_\_}$$

$$= \boxed{\qquad\qquad}$$

こうすれば高校の数学 III の内容になる.

定積分の部分積分法

☞ 部分積分法の場合には教科書の定理 10.4.1 の公式を使うと間違える可能性が非常に高い.
この場合には <u>先に不定積分を求める</u> のがよい.

【 例題 67 】

(1) $\displaystyle\int_0^1 xe^x\,dx$

$$\int xe^x\,dx = \boxed{}$$

したがって, $\displaystyle\int_0^1 xe^x\,dx = \Big[\,\boxed{}\,\Big]_0^1$

$$= \boxed{}$$

---

(2) $\displaystyle\int_0^{\frac{\pi}{3}} x\cos x\,dx$

$$\int x\cos x\,dx = \boxed{}$$

したがって, $\displaystyle\int_0^{\frac{\pi}{3}} x\cos x\,dx = \Big[\,\boxed{}\,\Big]_0^{\frac{\pi}{3}}$

$$= \boxed{}$$

(3) $\displaystyle\int_1^e \log x \, dx$

$$\int \log x \, dx = \boxed{}$$

したがって, $\displaystyle\int_1^e \log x \, dx = \Big[\boxed{} \Big]_1^e$

$$= \boxed{}$$