逆三角関数と積分【例題 56 参照】

$$\int \frac{dx}{\sqrt{1-x^2}} = \boxed{} \qquad \int \frac{dx}{1+x^2} = \boxed{}$$

(1) $\displaystyle\int \frac{dx}{\sqrt{1-(5x)^2}}$ $\qquad t = \boxed{}$ より

$$\int \frac{dx}{\sqrt{1-(5x)^2}} = \frac{1}{\boxed{}} \times \boxed{} = \boxed{}$$

(2) $\displaystyle\int \frac{dx}{\sqrt{1-(3x+1)^2}}$ $\qquad t = \boxed{}$ より

$$\int \frac{dx}{\sqrt{1-(3x+1)^2}} = \frac{1}{\boxed{}} \times \boxed{} = \boxed{}$$

(3) $\displaystyle\int \frac{dx}{1+9x^2} = \boxed{}$ $\qquad t = \boxed{}$ より

$$\int \frac{dx}{1+9x^2} = \frac{1}{\boxed{}} \times \boxed{} = \boxed{}$$

(4) $\displaystyle\int \frac{dx}{1+(5x-7)^2}$ $\qquad t = \boxed{}$ より

$$\int \frac{dx}{1+(5x-7)^2} = \frac{1}{\boxed{}} \times \boxed{} = \boxed{}$$

【置換積分を使った公式 2】

$$\int \frac{f'(x)}{f(x)}\,dx = \log|f(x)| + C$$

薬学ではよく使う公式

分数関数を積分する場合はこの公式に注意する．【練習問題 47 参照】

(1) $\displaystyle\int \frac{x}{x^2+1}\,dx$

$(x^2+1)' = \boxed{}$ より $\displaystyle\frac{x}{x^2+1} = \boxed{} \times \frac{(x^2+1)'}{x^2+1}$

したがって，　$\displaystyle\int \frac{x}{x^2+1}\,dx = \boxed{} \times \log \boxed{}$

(2) $\displaystyle\int \frac{x^2+2x}{x^3+3x^2+1}\,dx$

$(x^3+3x^2+1)' = \boxed{}$ より

$$\frac{x^2+2x}{x^3+3x^2+1} = \boxed{} \times \frac{(x^3+3x^2+1)'}{x^3+3x^2+1}$$

したがって，　$\displaystyle\int \frac{x^2+2x}{x^3+3x^2+1}\,dx = \boxed{} \times \log \boxed{}$

次の積分を求めよ．

(3) $\displaystyle\int \frac{x}{3x^2+5}\,dx$　　(4) $\displaystyle\int \frac{\cos x}{\sin x}\,dx$

【注意】例題 57 はできなくて良い (公式とみなす)．

専門で使うこともあるので，教科書に載っていることを覚えていれば良い．

部分積分の計算

ここが重要

$\displaystyle\int f(x)\,g'(x)\,dx$　から　$\displaystyle\int f'(x)\,g(x)\,dx$ が計算可能になるように変形する.

$$\begin{cases} f(x) = \boxed{\quad ❷ \quad} \\ g'(x) = \boxed{\quad ❸ \quad} \end{cases} \text{とおくと} \begin{cases} f'(x) = \boxed{\quad ❶ \quad \text{簡単な式}} \\ g(x) = \boxed{\quad ❹ \quad} \end{cases}$$

$$\begin{aligned} \int f(x)\ g'(x)\,dx &= \underset{\boxed{\quad ❷ \quad}}{\overset{f(x)}{\parallel}} \quad \underset{\boxed{\quad ❹ \quad}}{\overset{g(x)}{\parallel}} \quad - \quad \underset{\int \boxed{\ ❸ \times ❹\ }\,dx}{\overset{\int f'(x)\ g(x)\,dx}{\parallel}} \\ &= \boxed{\qquad\qquad\qquad\qquad} \end{aligned}$$

部分積分の公式は複雑なので, 公式を書いてから計算する

【 例題 58 】

(1) $\displaystyle\int x\cos x\,dx$

$$\begin{cases} f(x) = \boxed{\qquad} \\ g'(x) = \boxed{\qquad} \end{cases} \text{とおくと} \begin{cases} f'(x) = \boxed{\qquad} \\ g(x) = \boxed{\qquad} \end{cases}$$

$$\begin{aligned} \int f(x)\ g'(x)\,dx &= \underset{\boxed{\qquad}}{\overset{f(x)}{\parallel}} \quad \underset{\boxed{\qquad}}{\overset{g(x)}{\parallel}} \quad - \quad \underset{\int \boxed{\qquad}\,dx}{\overset{\int f'(x)\ g(x)\,dx}{\parallel}} \\ &= \boxed{\qquad\qquad\qquad\qquad} \end{aligned}$$

(2) $\displaystyle\int xe^x\,dx$

$$\left\{\begin{array}{l} f(x) = \boxed{} \\ g'(x) = \boxed{} \end{array}\right. \quad \text{とおくと} \quad \left\{\begin{array}{l} f'(x) = \boxed{} \\ g(x) = \boxed{} \end{array}\right.$$

$$\begin{array}{rcccc} \displaystyle\int f(x)\ g'(x)\,dx = & f(x) & g(x) & - & \displaystyle\int f'(x)\ g(x)\,dx \\ & \parallel & \parallel & & \parallel \\ = & \boxed{} & \boxed{} & - & \displaystyle\int \boxed{}\,dx \end{array}$$

$$= \boxed{}$$

【 例題 59 】（１が　かくれんぼ ）

(1) $\displaystyle\int \log x\,dx$

$$\left\{\begin{array}{l} f(x) = \boxed{} \\ g'(x) = \boxed{} \end{array}\right. \quad \text{とおくと} \quad \left\{\begin{array}{l} f'(x) = \boxed{} \\ g(x) = \boxed{} \end{array}\right.$$

$$\begin{array}{rcccc} \displaystyle\int f(x)\ g'(x)\,dx = & f(x) & g(x) & - & \displaystyle\int f'(x)\ g(x)\,dx \\ & \parallel & \parallel & & \parallel \\ = & \boxed{} & \boxed{} & - & \displaystyle\int \boxed{}\,dx \end{array}$$

$$= \boxed{}$$