部分積分の計算 **2012-10-5**

ここが重要

$\displaystyle\int f(x)\,g'(x)\,dx$ から $\displaystyle\int f'(x)\,g(x)\,dx$ が計算可能になるように変形する.

$$\begin{cases} f(x) = \boxed{\quad(1)\quad} \\ g'(x) = \boxed{\quad(2)\quad} \end{cases}$$

とおくと

$$\begin{cases} f'(x) = \boxed{\quad(3)\quad \text{簡単な式}} \\ g(x) = \boxed{\quad(4)\quad} \end{cases}$$

$$\int f(x)\ g'(x)\,dx = \underset{\boxed{(1)}}{\underset{\|}{f(x)}}\ \underset{\boxed{(4)}}{\underset{\|}{g(x)}} - \underset{\int \boxed{(3)\times(4)}\,dx}{\underset{\|}{\int f'(x)\ g(x)\,dx}}$$

$$= \boxed{(1)}\ \boxed{(4)} - \int \boxed{(3)\times(4)}\,dx$$

$$= \boxed{\qquad\qquad\qquad\qquad\qquad}$$

部分積分の公式は複雑なので, 公式を書いてから計算する

【 例題 58 】

(1) $\displaystyle\int x\cos x\,dx$

$$\begin{cases} f(x) = \boxed{} \\[1em] g'(x) = \boxed{} \end{cases} \text{とおくと} \quad \begin{cases} f'(x) = \boxed{} \\[1em] g(x) = \boxed{} \end{cases}$$

$$\begin{aligned} \int f(x)\ g'(x)\,dx &= \underset{\boxed{}}{\underset{\shortparallel}{f(x)}}\ \ \underset{\boxed{}}{\underset{\shortparallel}{g(x)}} - \underset{\boxed{}}{\underset{\shortparallel}{\int f'(x)\ g(x)\,dx}} \\ &= \boxed{}\ \boxed{} - \int \boxed{}\,dx \\ &= \boxed{} \end{aligned}$$

(2) $\displaystyle\int xe^x\,dx$

$$\begin{cases} f(x) = \boxed{} \\[1em] g'(x) = \boxed{} \end{cases} \text{とおくと} \quad \begin{cases} f'(x) = \boxed{} \\[1em] g(x) = \boxed{} \end{cases}$$

$$\begin{aligned} \int f(x)\ g'(x)\,dx &= \underset{\boxed{}}{\underset{\shortparallel}{f(x)}}\ \ \underset{\boxed{}}{\underset{\shortparallel}{g(x)}} - \underset{\boxed{}}{\underset{\shortparallel}{\int f'(x)\ g(x)\,dx}} \\ &= \boxed{}\ \boxed{} - \int \boxed{}\,dx \\ &= \boxed{} \end{aligned}$$

【 例題 59 】（1 が　かくれんぼ ）

(1) $\int \log x \, dx$

$$\begin{cases} f(x) = \boxed{\qquad\qquad} \\ g'(x) = \boxed{\qquad\qquad} \end{cases} \text{とおくと} \quad \begin{cases} f'(x) = \boxed{\qquad\qquad} \\ g(x) = \boxed{\qquad\qquad} \end{cases}$$

$$\begin{array}{rcccl} \displaystyle\int f(x)\ g'(x)\,dx = & f(x) & g(x) & - & \displaystyle\int f'(x)\ g(x)\,dx \\ & \| & \| & & \qquad\| \\ = & \boxed{\qquad\qquad} & \boxed{\qquad\qquad} & - & \displaystyle\int \boxed{\qquad\qquad}\,dx \\ = & \boxed{\qquad\qquad\qquad\qquad\qquad\qquad} & & & \end{array}$$

【 練習問題 49 】（1 が　かくれんぼ ）

$\int \mathrm{Tan}^{-1} x \, dx$

$$\begin{cases} f(x) = \boxed{\qquad\qquad} \\ g'(x) = \boxed{\qquad\qquad} \end{cases} \text{とおくと} \quad \begin{cases} f'(x) = \boxed{\qquad\qquad} \\ g(x) = \boxed{\qquad\qquad} \end{cases}$$

$$\begin{array}{rcccl} \displaystyle\int f(x)\ g'(x)\,dx = & f(x) & g(x) & - & \displaystyle\int f'(x)\ g(x)\,dx \\ & \| & \| & & \qquad\| \\ = & \boxed{\qquad\qquad} & \boxed{\qquad\qquad} & - & \displaystyle\int \boxed{\qquad\qquad}\,dx \\ = & \boxed{\qquad\qquad\qquad\qquad\qquad\qquad} & & & \end{array}$$

部分分数分解

$$\frac{1}{(x+1)(x+2)} = \frac{\boxed{}}{x+1} + \frac{\boxed{}}{x+2}$$

$$\frac{x+4}{(2x+1)(x-3)} = \frac{\boxed{}}{2x+1} + \frac{\boxed{}}{x-3}$$

$$\frac{3x+2}{(x+3)(x-4)} = \frac{\boxed{}}{x+3} + \frac{\boxed{}}{x-4}$$

$$\frac{x}{(x+1)^2} = \frac{\boxed{}}{x+1} + \frac{\boxed{}}{(x+1)^2}$$

$$\frac{3x^3}{x^2-1} = \boxed{} + \frac{\boxed{}}{x+1} + \frac{\boxed{}}{x-1}$$

有理化

$$\frac{1}{\sqrt{x+1}+\sqrt{x}} = \frac{1}{\sqrt{x+1}+\sqrt{x}} \times \boxed{} = \boxed{}$$

$$\frac{1}{\sqrt{x+1}-\sqrt{x+3}} = \frac{1}{\sqrt{x+1}-\sqrt{x+3}} \times \boxed{} = \boxed{}$$